<u>MOL solution of PDEs</u>

Computer Project (due on Tuesday, June 28, 2011)

Problem. Consider a problem arising in the modeling of flame propagation that involves the system of PDEs

$$\frac{\partial u}{\partial t} + f(u,v) = \frac{\partial^2 u}{\partial x^2},$$
$$\frac{\partial v}{\partial t} - f(u,v) = \frac{\partial^2 v}{\partial x^2}, \tag{1}$$

where $f(u,v) = 3.52 \times 10^6 \ u \ e^{-4/v}$ and $0 \le x \le 1$. The boundary conditions are

$$u_x(0,t) = 0, \qquad u_x(1,t) = 0,$$
$$v_x(0,t) = 0, \qquad v(1,t) = 1.2,$$

and the initial conditions are

$$u(x,0) = 1, \quad v(x,0) = 1.2 + \tanh(1000(x-1)).$$

Here, $u(x,t)$ and $v(x,t)$ correspond to mass density and temperature, respectively. A constant value for the temperature at the right boundary models a heat source which generates a steep flame front. The front propagates from right to left at a relatively high velocity and reaches the left boundary slightly after $T = 0.006$.

Numerical method. Use the code (`movfdm.m`) for the moving finite difference method which uses central finite differences for spatial discretization and the Method of Lines (MOL) approach for solving systems of partial differential equations. It can be used for computation with a fixed uniform mesh or an adaptive moving mesh. The code can be downloaded at

`http://www.math.ku.edu/~huang/research/matlab-codes/matlabcodes.html`

An example driver for Burgers' equation (`sec1_4_burgersFDM.m`) can also be found in the directory.

Assignment:

1. Modify `sec1_4_burgersFDM.m` for the above IBVP. Consider to output the solution at $t = 0.001, 0.002, 0.003, 0.004, 0.005$ and 0.006.
2. This IBVP does not have an exact solution. You may solve the problem on a very fine, uniform mesh (say $J = 1000$) and use the obtained solution as the "reference" solution.
3. Carry out computations on fixed meshes of several values of J.
4. Carry out computations on moving meshes of several values of J.
5. Plot the computed solutions (for both u and v) as function of x for several time instants.
6. Turn in your reports with solution plots and a copy of your program. Discuss what you have seen and why you believe your solutions are correct.
7. Collaboration among teams of two or three students is encouraged, with all participants receiving the same credit. Each team just needs to turn in one report with a list of all participants.