Lecture 8

More general BCs and Linear problems
§2.13 and §2.15

Generizations:

1°. More general BCs and linear problems

2°. Conservations and symmetries

3°. nonlinear equations

4°. Singular Problems

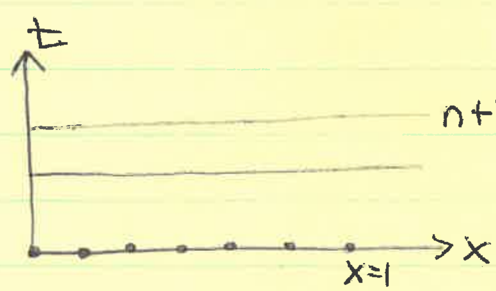
5°. mesh adaptation/movement

Part I BCs

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} & x \in (0,1),\ t>0 \\ u(x,0) = u^0(x) & \\ \dfrac{\partial u}{\partial x}(0,t) = \alpha(t)u(0,t) + g(t) & t>0 \\ u(1,t) = 0 & \end{cases}$$

Method 1 (Direct 1st Order)

$x_j = j\Delta x$, $\Delta x = \frac{1}{J}$,



$$\frac{U_1^{n+1} - U_0^{n+1}}{\Delta x} = \alpha(t_{n+1}) U_0^{n+1} + g(t_{n+1})$$

based on (x_0, t_{n+1})

$$T_0^{n+1} = O(\Delta x)$$

Method 2 (Fictitious point, widely used)

$x_j = j\Delta x$, $j = 0, 1, \cdots, J$

introduce $x_{-1} = -\Delta x$, U_{-1} (fictitious point) based on (x_0, t_{n+1}):

$$\frac{U_{+1}^{n+1} - U_{-1}^{n+1}}{2\Delta x} = \alpha(t_{n+1}) U_0^{n+1} + g(t_{n+1}) \quad \text{(BC)}$$

$$\frac{U_0^{n+1} - U_0^{n}}{\Delta t} = \frac{1}{\Delta x^2}\left[U_1^n - 2U_0^n + U_{-1}^n\right]$$

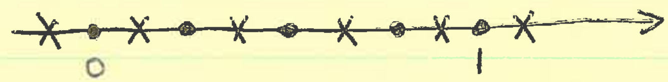
Method 3 (Staggered grid, widely used)

$$x_j = j\Delta x \qquad j = 0, \cdots, J-$$

$$x_{j+1/2} = (j+\tfrac{1}{2})\Delta x, \qquad j = -1, 0, \cdots, J$$

$$U_{j+1/2}^{n+1} \qquad (j = -1, 0, 1, \cdots, J)$$

interior points:



$$\frac{U_{j+1/2}^{n+1} - U_{j+1/2}^{n}}{\Delta t} = \frac{1}{\Delta x^2}\left[U_{j+3/2}^n - 2U_{j+1/2}^n + U_{j-1/2}^n\right]$$

based on $(x_{j+1/2}, t_n)$ $\qquad j = 0, 1, \cdots, J-1$

$$\frac{U_{1/2}^{n+1} - U_{-1/2}^{n+1}}{\Delta x} = \alpha(t_{n+1}) \frac{U_{1/2}^{n+1} + U_{-1/2}}{2} + g(t_{n+1})$$

$$\frac{U_{J+1/2}^{n+1} + U_{J-1/2}^{n+1}}{2} = 0$$

Part II

$$\frac{\partial u}{\partial t} = a(x,t)\frac{\partial^2 u}{\partial x^2} + b(x,t)\frac{\partial u}{\partial x} + c(x,t)u + d(x,t)$$

- the point based on
- explicit, Crank-Nicolson, implict.
- stability, truncation error, convergence.